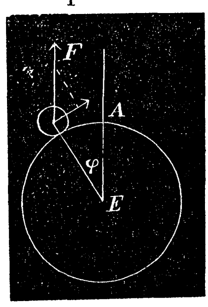
ON THE PHENOMENA OF THE TIDES.

BY PROF. ELIAS SCHNEIDER, MILTON, PA.

IN Vol. VI, No. 6 of the ANALYST, a problem, No. 281, was proposed for solution. The solution was commenced in No. 2 and completed in No. 3, Vol. 7. The problem was given with the expectation that its solution would throw additional light on a subject, in the treatment of which our books do not, in my opinion, include all the facts relating to it. It is now proposed to show how the solution gives this additional light.

According to the result obtained in the solution, the ball would pass a point 180° from the initial point, nearly 3 hours before the point A will pass the same point in space. If the gravitating power of the sun has such an effect upon a smooth solid body resting on an equally smooth surface of the earth, it must also have a similar effect upon a fluid, resting on the earth's surface. Suppose, then, that instead of a ball situated as stated in the problem, the earth be surrounded by a belt of water, about 2000 m's wide and 5 miles deep. Now suppose the earth to rotate on an axis once in 24 hours, the water and earth being subject to no influence but that of their mutual attraction. Then every particle of this water will be relatively at rest with respect to the earth.

But now suppose the sun to come in to position and to act upon the water at A in the direction of the diameter AE. When, from the rotation of the earth, the diameter AE ceases to point towards the sun the particles of water at A will cease to be in equilibrium, and will move over the earth's surface toward the attracting body; and, according to the solution referred to, the water at A would acquire such an increased velocity as to pass a point 180° from the initial point, nearly 3 hours before the point A will pass the same point in space. Therefore, on account of increased centrifugal force, there will be a tendency in the water to move along a line tangent to the earth's surface at D, causing the water to swell out there. In other words, there will be a tide wave at D. This takes place accord- to the statement and solution of the problem, under the supposition that the earth has no other motion than rotation upon an axis. It follows therefore, that, if the earth had no other motion than axial rotation, the tide wave would be produced by the same cause that is supposed to move the ball, and that there would be no necessity for seeking the cause of the phenomena of the solar tide in the power of the sun to *lift* the water above its gen'l level,



and that the occurrence of this tide two or three hours after meridian wo'd not be due to the resistance made by the inertia of the water.

But the earth, in addition to an axial rotation, has also an orbital motion around the sun. Would this motion, if recognized in the prob. 281, change materially the motion of the ball? * * * *

[We proceed to answer Prof. Schneider's question as here proposed; but desire, first, to say that we dissent entirely to his conclusions deduced above.

In entering this protest, it is sufficient to say that the water in the supposed belt is *not* free to move forward as the ball is, because a limited portion of the waters of the sea cannot advance horizontally without first displacing an equal quantity of water, and the entire "belt" cannot move around the earth because the attraction upon the water on *both sides* of the diameter AD is in the same direction. Moreover, *analysis*, which is our only *safe* guide, leads to entirely different results in the two cases.

In reply to the above inquiry, we answer, it would. If we assume in prob. 281 that the earth has also an orbital motion around the sun, the ball, when at the initial point A, will also have an orbital motion, the centrifugal force from which will not only equal the centripetal force F, but will slightly exceed it for all points on the surface more remote from the sun than E (the center of the earth), and therefore the tendency to motion in the ball, while it remains within the first quadrant from the point A, in space, will be backward, or *from* the sun; for the only force that tends to roll the ball is the *difference* between its centripetal and centrifugal forces, which, in this case may be regarded as *negative*, the positive direction being toward the sun.

When, from the motion of the earth on its axis, the ball shall be transferred to a point in space which is nearer the sun than the point E, the difference between the centripetal and centrifugal forces will become *positive* and the ball will tend to move forward or toward the sun.

It follows therefore that matter, which is subject to the attraction of a remote body, and which is free to move horizontally over the earth's surface, will tend to accumulate both *under* and *opposite* the attracting body. This conclusion is in harmony with the theory of the tides, as it should be; for though the waters are *not* free to move horizontally, their equilibrium is continually disturbed by the attracting body, the distortion tending to a like accumulation both under and opposite the attracting body.

Because, in the solution of prob. 281, the earth is supposed to be stationary in its orbit, the centrifugal force developed by the rolling of the ball, being at right angles with its motion, need not be considered; and because the *difference* of the attracting force of the sun, for different positions of the ball, is small in comparison with the attracting force at any particular point,

the force which tends to roll the ball is regarded as constant, which renders the solution comparatively simple.

But in discussing the equilibrium of the sea, subject to a like attraction by the sun while the earth is moving in its orbit, the disturbing force is obviously the *difference* between the centripetal and centrifugal forces at any particular point, and hence is a *variable* force; therefore an equation involving the theory of the tides is much more complicated than that representing the solution of problem 281.

On revising his numerical calculation for the determination of t in the solution of 281, Mr. Kummell finds

$$t_\pi = 9^h 05^m 42^s.5.$$

Equation (9), p. 88, gives, by expanding in a series and integrating,

$$t_\pi = \frac{1.188}{2} 12^h \left[1 + \frac{1}{4}e^2 + \frac{9}{64}e^4 + \frac{25}{256}e^6 + \frac{1225}{16382}e^8 + \ldots\right].$$

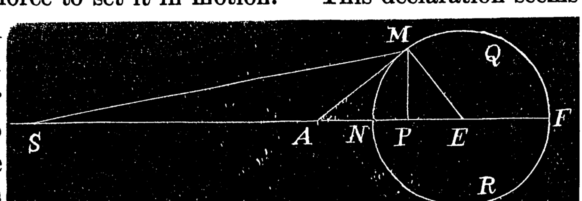
Summing twelve terms of this series we get

$t_\pi = 9^h 05^m 27^s$; being a little less than the answer obtained from eq. (10) in consequence of the slow convergence of this series.—Ed]

NOTE.—"Proof" of the preceding Art. and comments was sent to Prof. Schneider with the hope that the suppression of the latter part of his paper would be approved by him. As he insists, however, upon the printing of the entire paper and claims that we have omitted "the strongest part": in order that no injustice be done to the argument of Prof. Schneider, we here insert, without comment, that part of Prof. Schneider's paper which was omitted above.

* * * One of the computers says that, "the only *effect* of considering the progressive motion of the earth will be to shorten the time (say 2 minutes) of the ½ day required in the problem." It follows then, that the orbital motion of the earth does not effect materially the result of the solution, and therefore not the application.

The books tell us that "if the waters every where yielded immediately to the attractive force, it would always be high-water when the attractive force is on the meridian. But, owing to the inertia of the water, some time is necessary for so slight a force to set it in motion." This declaration seems to me contrary to actual facts. I will refer to an illustration in one of our books. "Suppose E to represent the centre of the earth, the circle FQN its



circumf., M a particle of water on the earth's surface, and S the sun. The entire earth being rigid, each part of it will move under the influence of the sun's attraction as if the whole were concentrated at its centre. But the attraction of the sun upon the particle M, being different from its mean attraction on the earth, will tend to make it move differently from the earth. The force which causes this difference of motion, as already explained, will be represented by the line MA. It is true that this same disturbing force is acting on that portion of the solid earth at M as well as upon the water. But the earth cannot yield on account of its rigidity; the water therefore tends to flow along the earth's surface from M towards N. There is therefore a residual force tending to make the water higher at N than at M." Now, is it not evident that the inertia of the water has already been overcome, before it reaches the point N? Its increased motion has commenced even before it reaches the point M, and this velocity is continually becoming more rapid until it has reached its maximum at N, and by virtue of its inertia it does not *resist* motion but *continues* moving in a line tang't to the earth at N, and thus causes the tide wave 2 or 3 hours after merid'n.

I have made reference to the solar tide only. The lunar tide is produced in the same way, the attracting power being the moon.

PROOF OF A PROPOSITION IN SOLID GEOMETRY.

BY PROF. E. W. HYDE, UNIVERSITY OF CINCINNATI.

It is probable that the prop sition given below has been previously stated. I do not remember, however, to have seen it, and it may be of interest to some of the readers of the ANALYST.

If two different surfaces of the same order are tangent to each other at every point in which they are pierced by a given right line, then the surfaces intersect each other in two *plane curves*, real or imaginary.

Let $V = 0$ and $W = 0$ be two planes passing through the given line, and let $U_1 = 0$, $U_2 = 0$, $U_3 = 0$ etc., be any surfaces whatever. Also let $\varphi(U_1, U_2 \text{ etc.})$ be any rational integral function of U_1, U_2 etc. Then the equations

$$\varphi(U_1,\ U_2 \text{ etc.}) + \lambda_1 V^2 + \mu_1 W^2 = 0, \qquad (1)$$

$$\varphi(U_1,\ U_2 \text{ etc.}) + \lambda_2 V^2 + \mu_2 W^2 = 0, \qquad (2)$$

represent two different surfaces each of which is tangent to $\varphi = 0$ at every point where it is pierced by the intersection of V and W; for (1) and (2) are each satisfied by such values of x, y, z as make φ, V and W simultaneously equal to zero; also, V and W being squared, the given line is the